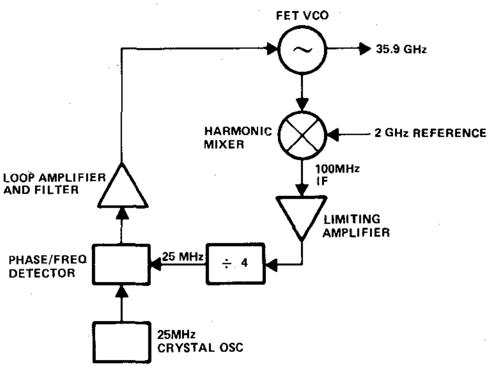

Fig. 10. Functional diagram of the phase-locked VCO.

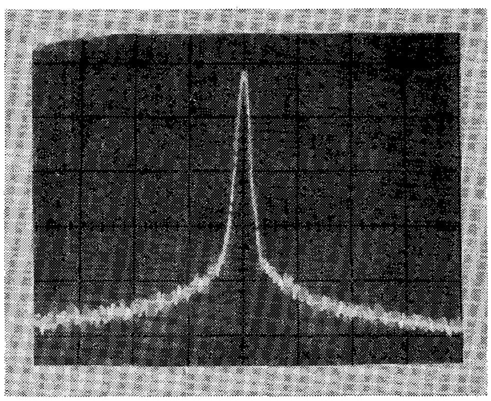

Fig. 11. Output spectrum of an FET oscillator, phase locked to a crystal oscillator. Res. BW = 1 KHz. Hor. Div = 10 KHz. Vert. Div = 10 dB.

ity, it was decided to phase lock one of the oscillators, S. no. 4 in Table I, to a crystal-controlled signal. The oscillator power sample was taken with a coaxial probe mounted on the cover of the VCO housing. A harmonic mixer [3] driven by a stable 2-GHz LO was used to obtain an IF frequency of 100 MHz, the FET oscillator frequency being 35.9 GHz. This IF was locked to a crystal oscillator with the help of a phase-locked loop, as shown in Fig. 10. The resulting spectrum as observed on a spectrum analyzer is shown in Fig. 11.

VI. Conclusion

Ka-band oscillators employing widely available GaAs FET's have been demonstrated. These offer advantages of lower power consumption and potentially lower cost in comparison with Gunn oscillators.

Acknowledgment

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Integral Transforms Useful for the Accelerated Summation of Periodic, Free-Space Green's Functions

R. LAMPE, P. KLOCK, SENIOR MEMBER, IEEE, AND P. MAYES, FELLOW, IEEE

***Abstract* —The Poisson summation formulas for two- and three-dimensional, periodic, free-space Green's functions of the Helmholtz and Laplace equations are cataloged in this paper. It is shown how these formulas can be applied for the efficient, approximate summation of series which arise in the computation of fields due to an infinite array of charge or current sources. The technique for approximating the summation of the series is valid for all arguments of a Green's function, even those which correspond to the region near a source singularity.**

I. Introduction

The computation of fields due to periodic sources can arise in the application of image theory when multiple ground planes are present. When the ground planes are closely spaced, the contribution to the field due to direct summation of the image sources is impractical due to very slow convergence. Hence, the image technique has been of limited usefulness in solving problems such as those involving striplines or cavities.

The Poisson summation formula [1] can sometimes be use to convert a slowly converging series into a rapidly converging one by allowing the series to be summed in the Fourier transform domain. To date, the Poisson summation formula has, when applied to periodic sources, been primarily applied to cases of determining fields at a distance from a source region. If the terms of a spatial domain series represent sampling of a function which is singular or nearly singular, such as the field near a source, the summation in the transform domain could be as slowly convergent as that in the spatial domain. Typically, the determination of a charge or current distribution for an element in a periodic array involves the evaluation of the field near a source singularity precluding useful application of the Poisson summation formula.

II. Series Acceleration Technique

A technique has been described for accelerating the summation of periodic, free-space Green's functions which circumvents the difficulty of slow convergence near a source singularity [2]. In its most basic form, this technique begins with two periodic functions: one for which a sum is required, and another which is asymptotically equal to the first but is smooth everywhere. These two functions will be defined as

$$\sum_{n=-\infty}^{\infty} f(n) \text{ and } \sum_{n=-\infty}^{\infty} g(n)$$

respectively. The equation which represents this technique is derived by combining Kummer's transformation [3] and the Poisson summation formula to obtain the following approximation:

$$\sum_{n=-\infty}^{\infty} f(n) \approx \sum_{n=-K}^{K} [f(n)-g(n)] + \sum_{n=-\infty}^{\infty} G(2\pi n) \quad (1)$$

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R. Lampe is with RCA, Missile and Surface Radar, Moorestown, NJ 08057.

P. Klock and P. Mayes are with the Department of Electrical Engineering, University of Illinois, Urbana, Illinois 61801.

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TABLE I
FREE-SPACE GREEN'S FUNCTIONS AND THEIR SUMMATION FORMULAS

1) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi}\left[(x-x')^2+(y-y')^2+(z-nd)^2\right]^{-1/2} \cdot \exp\left(-jk\left[(x-x')^2+(y-y')^2+(z-nd)^2\right]^{1/2}\right)$$
$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2\pi d}\right) K_0\left(\left[\left(\frac{2\pi n}{d}\right)^2 - k^2\right]^{1/2}\left[(x-x')^2+(y-y')^2\right]^{1/2}\right)\exp\left(\frac{-j2\pi nz}{d}\right)$$

2) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{1}{4j} H_0^{(2)}\left(k\left[(x-x')^2+(y-nd)^2\right]^{1/2}\right)$$
$$= \sum_{n=-\infty}^{\infty} \frac{1}{2d}\left[\left(\frac{2\pi n}{d}\right)^2 - k^2\right]^{-1/2} \exp\left(-\left[(x-x')^2\right]^{1/2}\left[\left(\frac{2\pi n}{d}\right)^2 - k^2\right]^{1/2}\right)\exp\left(\frac{-j2\pi ny}{d}\right)$$

3) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi}\left[(x-x')^2+(y-y')^2+(z-nd)^2\right]^{-1/2}$$
$$= \sum_{n=-\infty}^{\infty} \frac{1}{2\pi d} K_0\left(\left[(x-x')^2+(y-y')^2\right]^{1/2}\left|\frac{2\pi n}{d}\right|\right)\exp\left(\frac{-j2\pi nz}{d}\right)$$

4) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{-1}{2\pi}\ln\left(\left[(x-x')^2+(y-nd)^2\right]^{1/2}\right) = \sum_{n=-\infty}^{\infty} \frac{1}{2d}\left|\frac{2\pi n}{d}\right|^{-1}\exp\left(-\left[(x-x')^2\right]^{1/2}\left|\frac{2\pi n}{d}\right|\right)\exp\left(\frac{-j2\pi ny}{d}\right)$$

5) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{1}{2k}\exp(-k|x-nd|) = \sum_{n=-\infty}^{\infty} \frac{1}{d}\left[\left(\frac{2\pi n}{d}\right)^2 + k^2\right]^{-1}\exp\left(\frac{-j2\pi nx}{d}\right) = \frac{1}{2k}\cosh\left[\left(\frac{d}{2}-x\right)(k)\right]/\sinh\left(\frac{kd}{2}\right)$$

6) $$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} \frac{1}{j2k}\exp(-jk|x-nd|) = \sum_{n=-\infty}^{\infty} \frac{1}{d}\left[\left(\frac{2\pi n}{d}\right)^2 - k^2\right]^{-1}\exp\left(\frac{-j2\pi nx}{d}\right) = \frac{-1}{2k}\cos\left[\left(\frac{d}{2}-x\right)(k)\right]/\sin\left(\frac{kd}{2}\right)$$

where

$$\sum_{n=-\infty}^{\infty} G(2\pi n)$$

is the Poisson summation formula for the series

$$\sum_{n=-\infty}^{\infty} g(n).$$

Since the series

$$\sum_{n=-\infty}^{\infty} g(n)$$

represents a smooth function sampled periodically, its summation in the transform domain is greatly accelerated. Thus, this technique sums the contribution of the terms in a region near a singularity in the spatial domain and then sums the contribution of the slowly converging asymptotic terms in the transform domain for great savings in computation time.

This series acceleration technique has application in problems in electromagnetics which involve a periodic, free-space Green's function, such as in problems to which the method of images is applied, or those which deal with large arrays of antenna elements. In these problems, the Green's function of an integral equation can be identified as the function $f(n)$ in (1), and the functions $g(n)$ and $G(2\pi n)$ are determined from $f(n)$ and its Poisson summation formula by a simple substitution. For applying the acceleration technique, a complete list of periodic, free-space Green's functions of the Helmholtz and Laplace equations along with their Poisson summation formulas is compiled in Table I. These formulas permit efficient, approximate evaluation of the contribution to the field due to single or multiple summations of charge or current sources in rectangular coordinates. These formulas have in some cases been collected [4],[5] and in others derived by the authors. The notation used in Table I is as follows:

$K_0(x)$	modified Bessel function of zeroth order,
$H_0^{(2)}(x)$	Hankel function of the second kind, zeroth order,
$\exp(x)$	exponential function,
$\ln(x)$	natural logarithm,
k	propagation constant of the medium,
j	square root of minus one.

A detailed description of how to recover $g(n)$ and $G(2\pi n)$ from $f(n)$ for use in the acceleration formula (1) follows. First, locate the appropriate Green's function $f(n)$ in Table I. If $f(n)$ involves formulas 1)–4) of Table I, then $g(n)$ and $G(2\pi n)$ are derived by substituting $(x-x')^2 + c^2$ for $(x-x')^2$ in the equation for $f(n)$ and its Poisson summation formula, respectively. In this substitution, c is a real constant chosen to minimize error and maximize computational efficiency. Formulas 5) and 6) of Table I represent one-dimensional Green's functions and their Poisson summation formulas. Since these one-dimensional Green's functions do not have source singularities, they can be readily summed by direct application of the Poisson summation formula; that is, the acceleration technique defined by (1) is not required. The Poisson summation formulas for these one-dimensional Green's functions are normally only needed in certain cases involving multiple summations.

Whereas the formulas in Table I are directly applicable to one-dimensional arrays of point and line sources, they can be easily extended, by successive application, to arrays of higher dimensions involving multiple summations. This extension is a result of the property that the Fourier transform of any of these Green's functions of any one dimension can be interpreted as the Green's function at the next lower dimension. For instance, the result of applying the Poisson summation formula one time to a two-dimensional array of point sources can be interpreted as a two-dimensional Green's function. The Poisson summation formula can then be applied again to recover the final Poisson summation formula for a two-dimensional array. This procedure is demonstrated by the following example.

To obtain a summation formula for a two-dimensional array of point current sources, the Poisson summation formula is first applied to the y coordinate of the three-dimensional Green's function yielding

$$f(p,q)=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\left[x^2+(y-pa)^2+(z-qb)^2\right]^{-1/2}\cdot\exp\left(-jk\left[x^2+(y-pa)^2+(z-qb)^2\right]^{1/2}\right)$$

$$=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{2\pi a}K_0\left(\left[\left(\frac{2\pi p}{a}\right)^2-k^2\right]^{1/2}\cdot\left[x^2+(z-qb)^2\right]^{1/2}\right)\exp\left(\frac{-j2\pi py}{a}\right). \tag{2}$$

An expression equivalent to a two-dimensional Green's function can be recovered by manipulation of the above expression giving

$$=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{a4j}H_0^{(2)}\left(\left[k^2-\left(\frac{2\pi p}{a}\right)^2\right]^{1/2}\cdot\left[x^2+(z-qb)^2\right]^{1/2}\right)\exp\left(\frac{-j2\pi py}{a}\right). \tag{3}$$

Applying the Poisson summation formula again, but this time to the z coordinate of (3), gives the following Poisson summation formula for the Green's function $f(p,q)$:

$$=\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{1}{2ab}\left[\left(\frac{2\pi q}{b}\right)^2+\left(\frac{2\pi p}{a}\right)^2-k^2\right]^{-1/2}\cdot\exp\left(-|x|\left[\left(\frac{2\pi q}{b}\right)^2+\left(\frac{2\pi p}{a}\right)^2-k^2\right]^{1/2}\right)\cdot\exp\left(\frac{-j2\pi py}{a}\right)\exp\left(\frac{-j2\pi qz}{b}\right). \tag{4}$$

The asymptotic form of the Green's function $g(p,q)$ and its Poisson summation formula $G(2\pi n, 2\pi q)$, required by the acceleration formula (1), are obtained from the Green's function and (4) by substituting (x^2+c^2) for x^2.

One final comment needs to be made. At first inspection, the singularity at $n=0$ in the Poisson summation formulas of the two- and three-dimensional Green's functions of the Laplace equation, i.e., formulas 3) and 4) in Table I, seem to cause trouble. In practice, the series Green's function can always be written as the difference of two functions, both of which having the functional form of $f(n)$ in either formula 3) or 4) of Table I. With the Green's function written in this form, the $n=0$ term of the Poisson summation formula equals zero, removing the singularity and obviating the problem.

III. Conclusion

The application of the series acceleration technique defined by (1) permits efficient computation of wide classes of problems which involve periodic sources. Many of these problems require integral transforms in the form of Poisson summation formulas which are not readily available. This paper presents a complete, convenient catalog of these Poisson summation formulas for Green's functions of the Helmholtz and Laplace equations which represent periodic sources in rectangular coordinates and homogeneous media.

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Constant-Frequency Synthesis of Lossy Microwave Two-Ports

LODEWIJK R. G. VERSFELD

Abstract —**At a fixed frequency, every linear time-invariant two-port can be described by its scattering matrix, whose elements represent eight real parameters. In this paper, it is proved that every lossy (linear, time-invariant) two-port can be canonically synthesized by eight "elementary" two-ports, which are characterized by one parameter only. Moreover, these elementary two-ports are passive and realizable in the microwave region. The synthesis is performed in the form of a cascade structure (with one "side arm" for the nonreciprocal case). Explicit formulas for the parameters of the elementary two-ports are derived.**

I. Introduction

This paper gives a "satisfactory" synthesis of linear, time-invariant, lossy microwave two-ports at a fixed frequency. It herewith solves part of the general problem of constant-frequency synthesis of microwave networks [1]. By "satisfactory" we mean a

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The author is with the Department of Electrical Engineering, Technische Hogeschool Eindhoven, 5600 MB, Eindhoven, The Netherlands.

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